Parking and Tamari-parking posets

Bérénice Delcroix-Oger
joint work with Clément Dupont (IMAG), Hélène Han (ENS Saclay), Matthieu Josuat-Vergès (IRIF) et Lucas Randazzo (Nomadic Labs)

q,t-Combinatorics in Algebra, Geometry and Topology
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Outline

1. From partition posets to parking posets
2. Parking posets (with M. Josuat-Vergès and L. Randazzo)
3. Tamari-parking posets (with M. Josuat-Vergès and H. Han)

From partition posets to parking posets

Outline

1. From partition posets to parking posets
   - The poset of noncrossing partitions
   - Noncrossing partitions and parking functions
2. Parking posets (with M. Josuat-Vergès and L. Randazzo)
3. Tamari-parking posets (with M. Josuat-Vergès and H. Han)

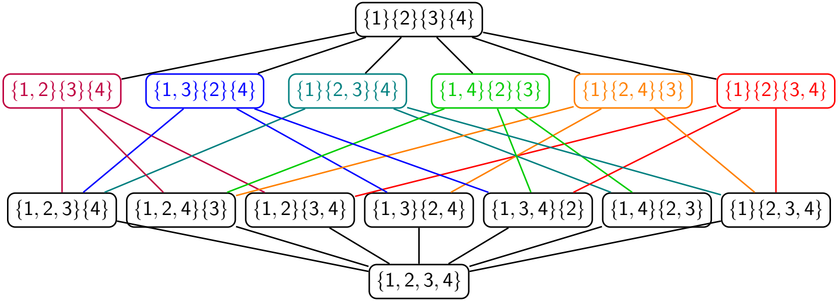
Posets(=partially ordered set) of (set) partitions $\Pi(V)$

Partitions of a set V :

$$\{V_1, \ldots, V_k\} \models V \Leftrightarrow V = \bigsqcup_{i=1}^{k} V_i,\, V_i \cap V_j = \emptyset \text{ for } i \neq j$$

Partial order on set partitions of a set V:

$$\{V_1, \ldots, V_k\} \leqslant \{V'_1, \ldots, V'_p\} \Leftrightarrow \forall i \in \{1, p\}, \exists j \in \{1, k\} \text{ s.t. } V'_i \subseteq V_j$$



What kind of combinatorics on posets ?

- Elements

For Π_n

Number of elements with k parts : $S(n,k)$ (Stirling number of the 2nd kind)
In terms of generating series : $\frac{1}{t}\exp\left(t(e^x-1)\right)-1=\sum_{n\geqslant 1}\sum_{\pi\in\Pi_n} t^{|\pi|-1}\frac{x^n}{n!}=:\mathbb{E}^+\circ t\mathbb{E}^+$.

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- Intervals

For Π_n

Number of intervals $[\pi,\tau]$ with π with k parts and τ with ℓ parts ($\ell\geqslant k$) :
$S(n,\ell+1)S(\ell+1,k+1)$
In terms of generating series : $\sum_{n\geqslant 1}\sum_{\pi\leqslant\tau\in\Pi_n} t^{|\pi|-1}q^{|\tau|-|\pi|}\frac{x^n}{n!}=\mathbb{E}^+\circ t\mathbb{E}^+\circ q\mathbb{E}^+$

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- Möbius numbers

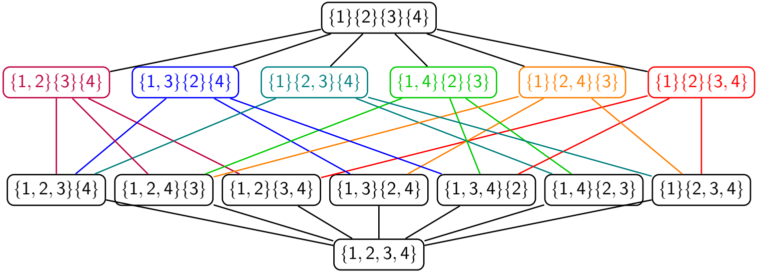
Möbius function and Möbius number

Definition

Möbius function of a poset P: $\mu : P \times P \to \mathbb{N}$ defined inductively by

$$\mu(x,x) = 1$$
$$\mu(x,y) = -\sum_{x \leqslant z < y} \mu(x,z) \text{ for } x < y.$$

Möbius number of a bounded poset P: $\mu(P) := \mu(\hat{0}, \hat{1})$.



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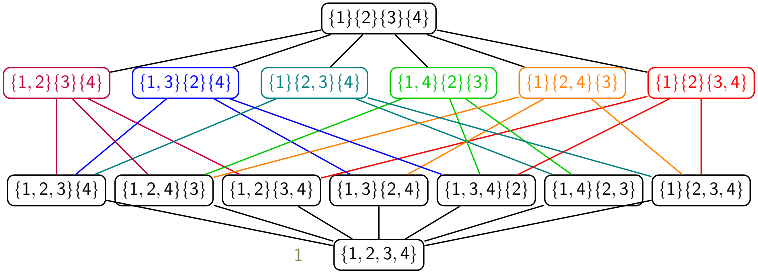

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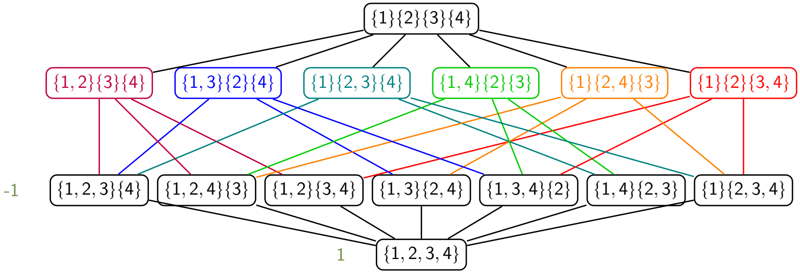

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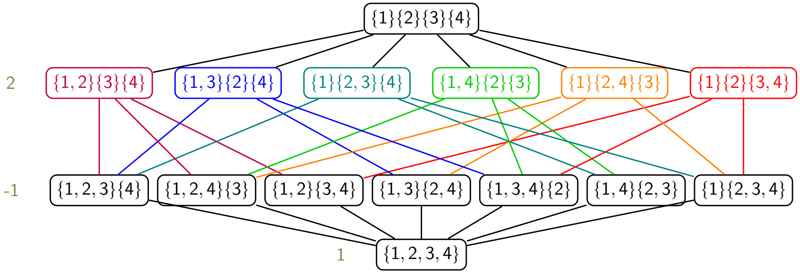

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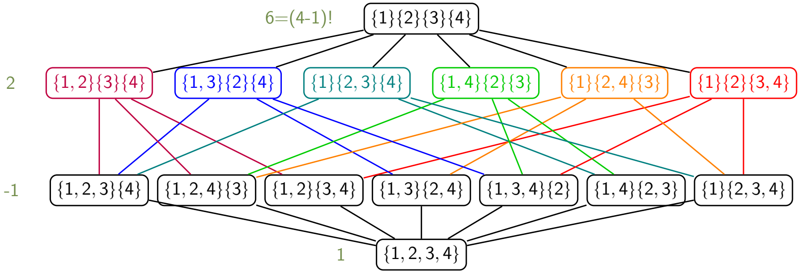

Whitney numbers of the first kind of Π_n

$$\sum_{n\geqslant 1}\sum_{\pi\in\Pi_n}\mu(\hat{0},\pi)t^{|\pi|-1}\frac{x^n}{n!}=\sum_{n\geqslant 1}\sum_{k=0}^{n-1}S(n,k+1)\times k!(-t)^k\frac{x^n}{n!}$$
$$=x+(1-t)\sum_{n\geqslant 2}f_{\mathrm{Perm}}(-t)\frac{x^n}{n!}$$

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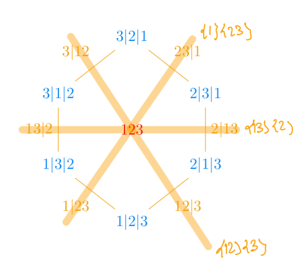


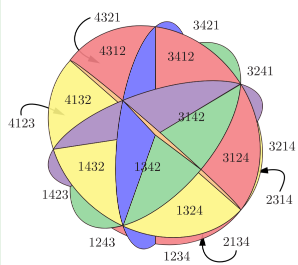

cf. "Cellular diagonals of permutahedra" with G. Laplante-Anfossi, V. Pilaud and K. Stoeckl (ArXiv : 2308.12119) (credit leftmost figure : V. Pilaud)

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In fact,

All given by counting chains !

Chains, Möbius numbers and Zeta polynomial

Let P be a poset with a unique minimum $\hat{0}$.

Definition (Stanley, 74)

The **zeta polynomial** of a poset P is the polynomial:

$$Z(P,k) = \sum_{\ell \geqslant 0} |\{a_1 < \ldots < a_k | a_1, \ldots, a_k \in P\}|.$$

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Proposition (Edelman, 80)

- *When P is bounded, $Z(P,-2) = \mu(P)$*
- *When P has only a minimum or a maximum, $Z(P,-1) = \mu(\hat{P})$*

Results on partition posets

We write Z_k for the e.g.f. of $Z(\Pi_n, k)$. Then,

$$Z_k = \mathbb{E}^+ \circ Z_{k-1}$$

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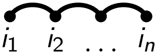
Z_0 &= \mathbb{E}^+ \\
Z_{-1} &= X \\
Z_{-2} &= \Sigma Lie = \bigoplus_{n \geqslant 1} Lie(n) \otimes_{\mathfrak{S}_n} sgn_n
\end{aligned}$$

Proposition (Haiman, Stanley, Joyal 80s ; Fresse 04)

$$h^{n-1}(\Pi_n) = Lie(n) \otimes_{\mathfrak{S}_n} sgn_n,$$

where $Lie(n)$ is the representation of the symmetric group corresponding to the Lie operad and sgn_n is the signature representation.

Noncrossing partitions [Kreweras, 1972]

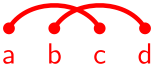
$\{i_1, \ldots, i_n\}$ with $i_1 < \ldots < i_n \rightarrow$ 

Definition (Kreweras, 1972)

A partition $\pi = \{\pi_1, \ldots, \pi_k\}$ of $\{1, \ldots, n\}$ is noncrossing iff

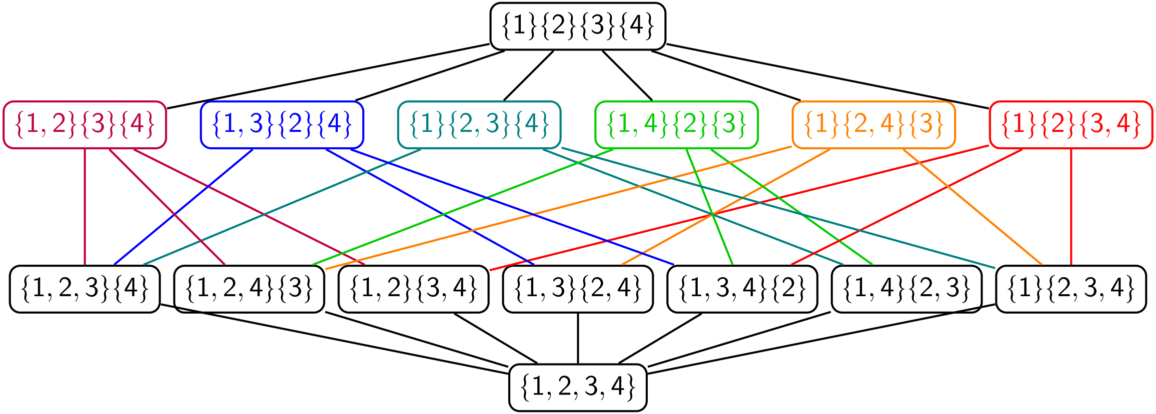
$$\begin{cases} a < b < c < d \\ a, c \in \pi_i \\ b, d \in \pi_j \end{cases} \implies i = j$$

NC_n = set of noncrossing partitions of $\{1, \ldots, n\}$



$\rightarrow$ counted by Catalan numbers $\frac{1}{n+1}\binom{2n}{n}$

Partition posets



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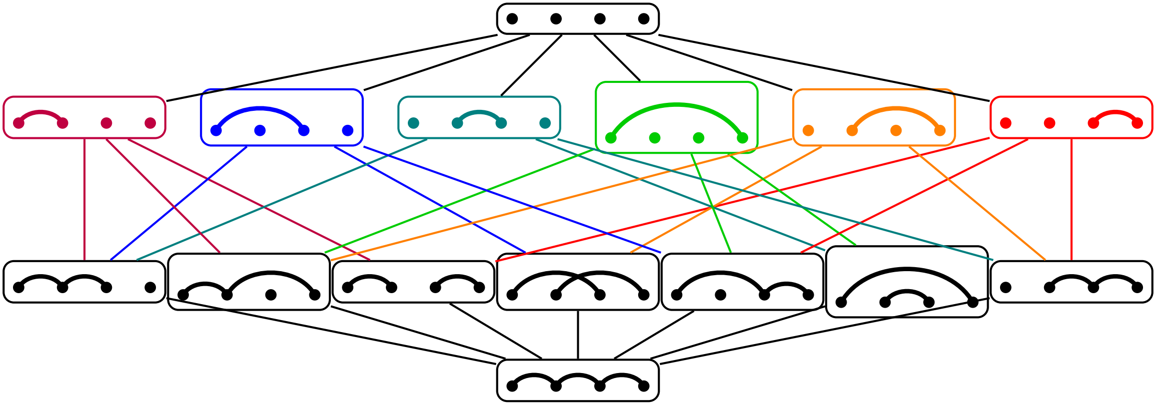

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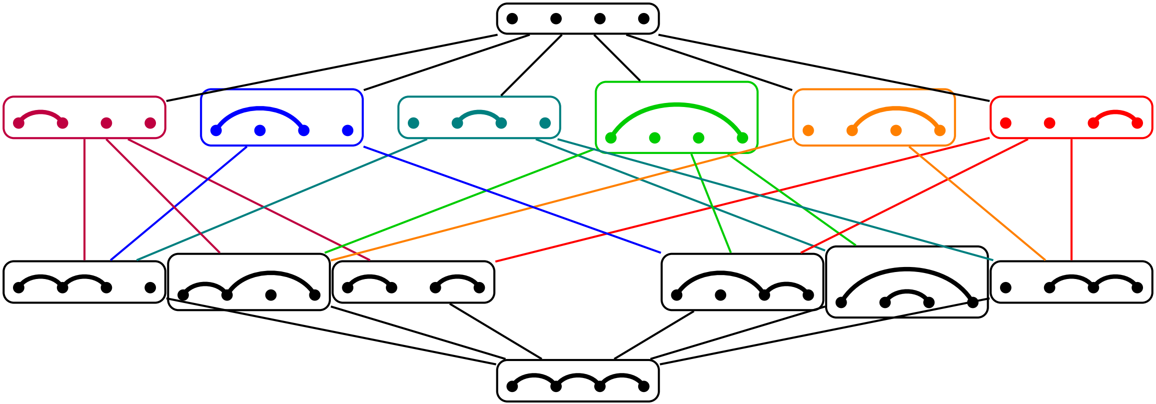

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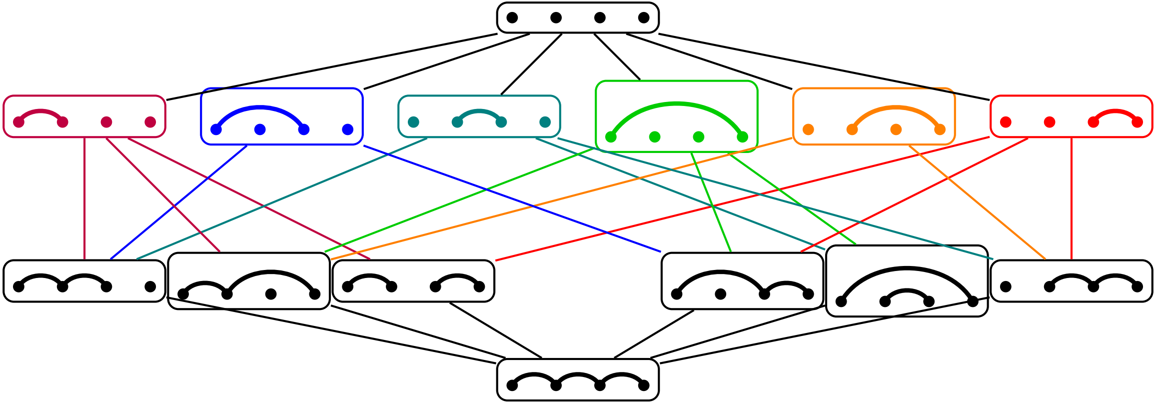


Problem

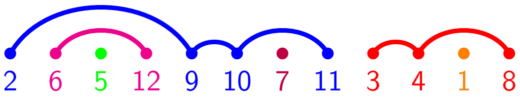
No action of the symmetric group on this poset !

Noncrossing 2-partitions

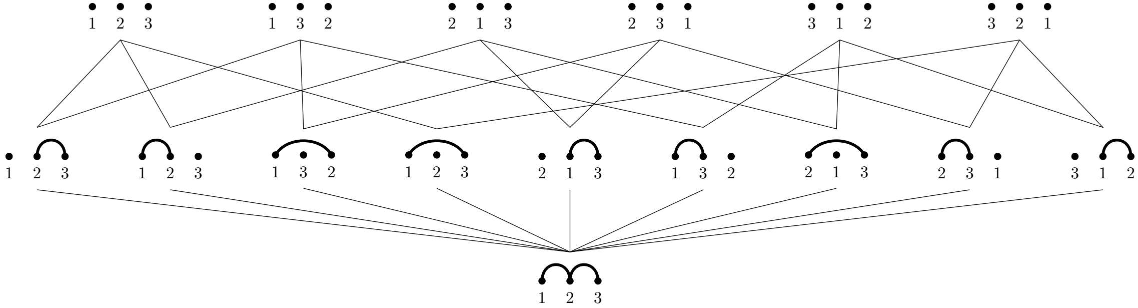
Definition (Edelman, 1980)

A n.c. 2-partition of size n is a pair $(\pi, \sigma) \in NCP_n \times \mathfrak{S}_n$ s.t.

$$\begin{cases} \{b_1, \ldots, b_k\} \in \pi \\ b_1 < b_2 < \ldots < b_k \end{cases} \implies \sigma(b_1) < \sigma(b_2) < \ldots < \sigma(b_k).$$



Noncrossing 2-partition poset



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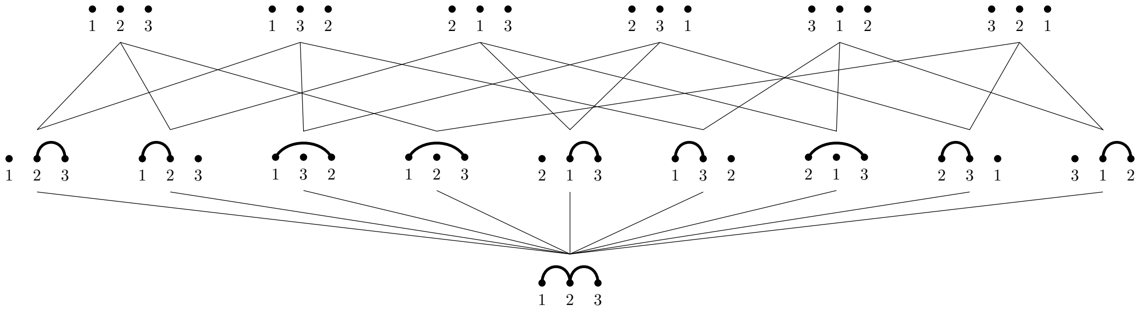

How many noncrossing 2-partitions are there ?

There are $(n+1)^{n-1}$ noncrossing 2-partitions, which is the same as the number of parking functions on n parking spots.

Let's drive and park !

1	2	3	4	5

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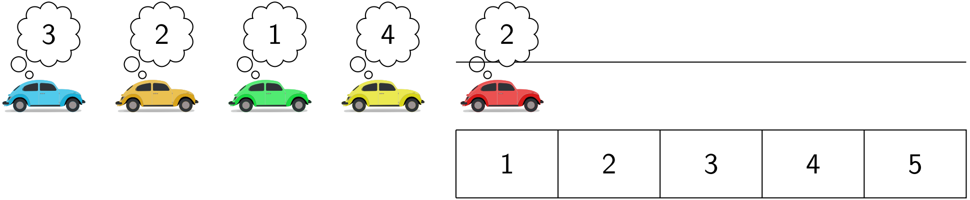

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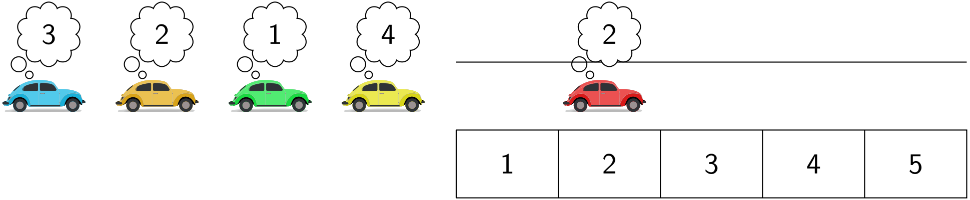

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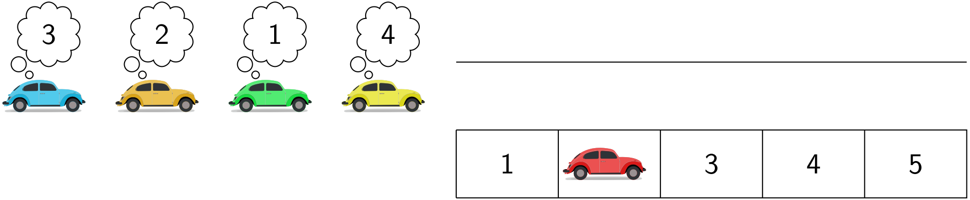

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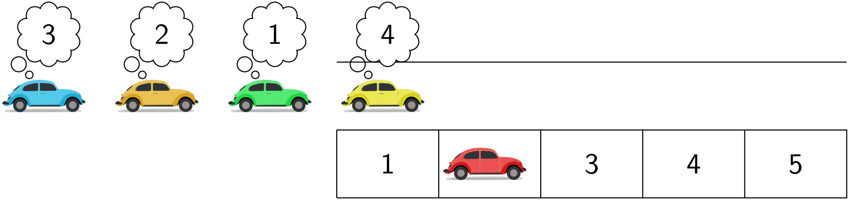

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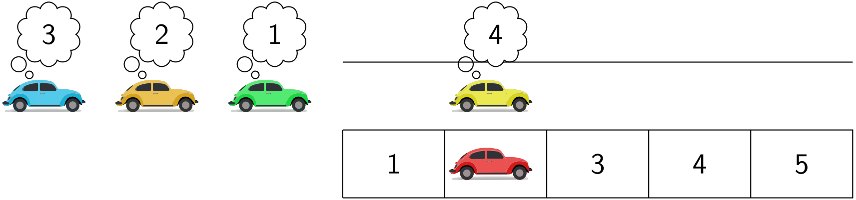

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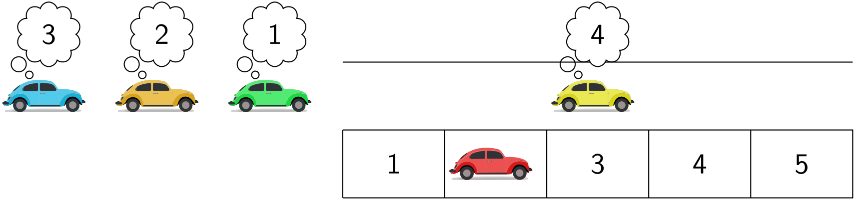

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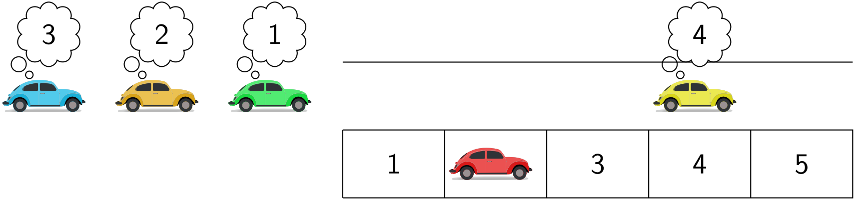

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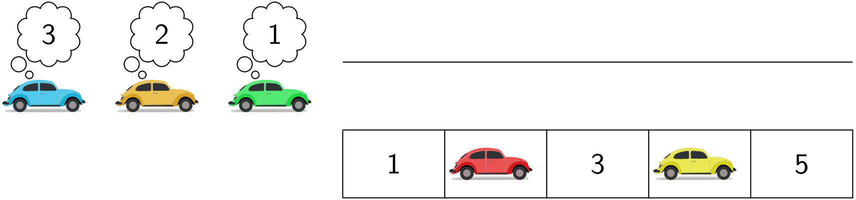

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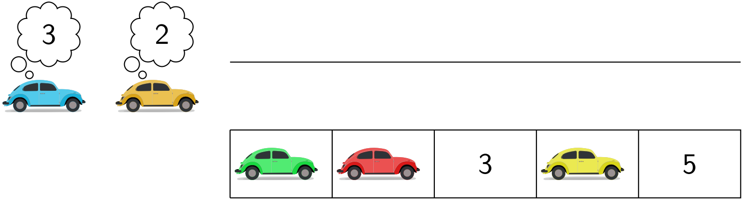

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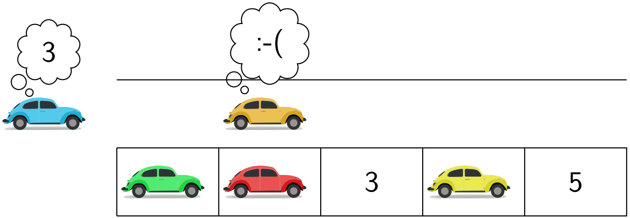

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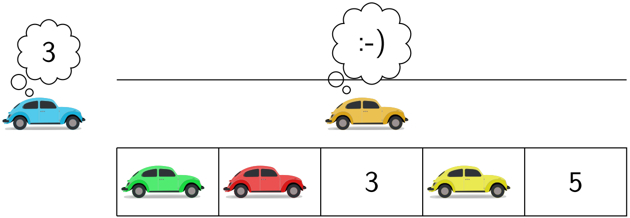

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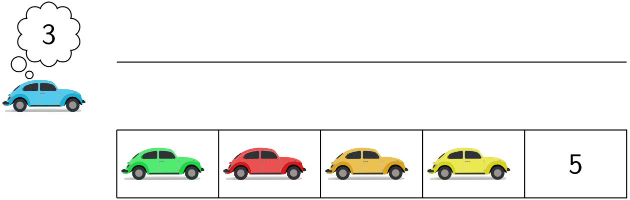

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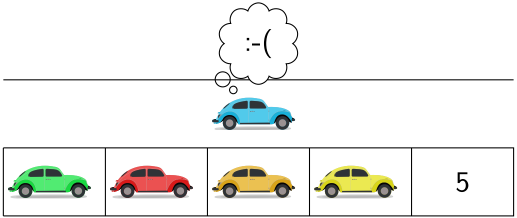

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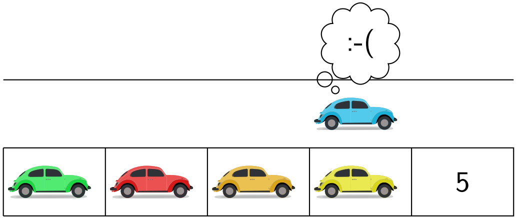

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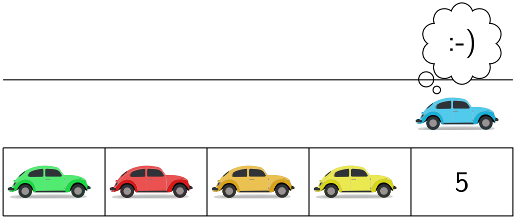

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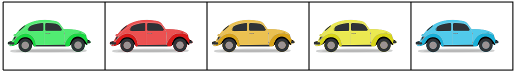


Definition

If every car can park, the word, obtained by reading from the first car entering the street to the last one, is called a parking function.

Examples and counter-examples:

Parking functions of length n	Other words of length n
1	
11,12,21	22
111, 112, 121, 211, 113, 131, 311,122, 212, 221, 123, 132, 213, 231, 312, 321	222, 333, 223, 232, 322, 233, 323, 332, 133, 313, 331

Formal definition of parking functions [Konheim-Weiss, 1966]

Definition

A sequence $\mathbf{a} = (a_1, \ldots, a_n) \in (\mathbb{N}^*)^n$ is a parking function of length n iff

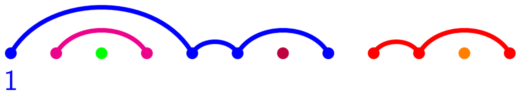
$$|\{i|1 \leqslant a_i \leqslant j\}| \geqslant j.$$

Denoting by $a^\uparrow$ the non-decreasing rearrangement of $\mathbf{a}$, this is equivalent to $1 \leqslant a_j < j$ for any $1 \leqslant j \leqslant n$. We call non-decreasing parking function a parking function satisfying $a = a^\uparrow$.

Theorem (Konheim-Weiss, 1966; Pollak, 1969)

There are $(n+1)^{n-1}$ parking functions of length n.
There are $C_n = \frac{1}{n+1}\binom{2n}{n}$ non-decreasing parking functions of length n.

Parking functions and noncrossing partitions



Label i by $\min \pi$ for $i \in \pi$

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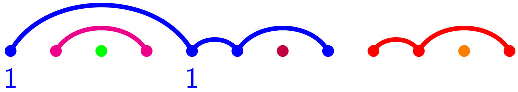

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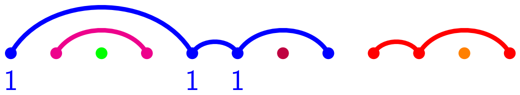

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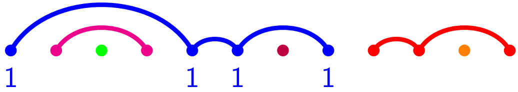

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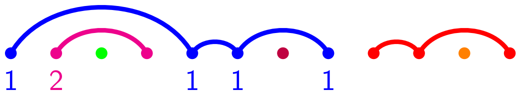

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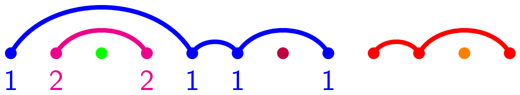

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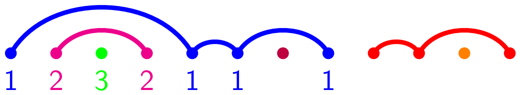
Parking functions and noncrossing partitions



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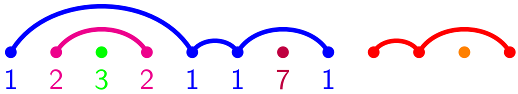

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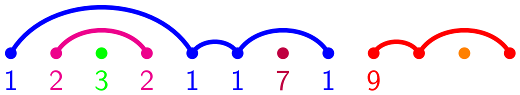

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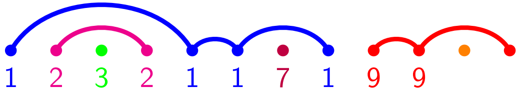

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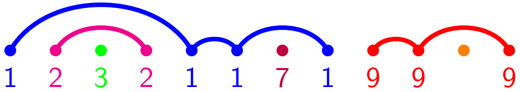

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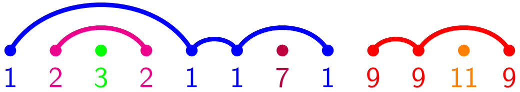

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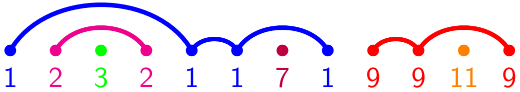

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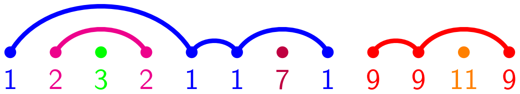
- Gives a parking function as the label of the jth node is smaller or equal to j.

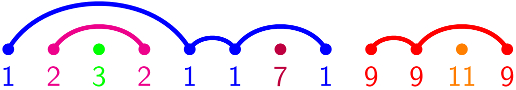
Parking functions and noncrossing partitions



Label i by $\min \pi$ for $i \in \pi$

- Gives a parking function as the label of the jth node is smaller or equal to j.
- It is the unique parking function in the orbit which maximizes the number of lucky cars. Call it the lucky parking function (used by Blass and Sagan to compute the Möbius function of Tamari lattices under the name "left bracket vector").

Posets of noncrossing 2-partitions in terms of parking functions

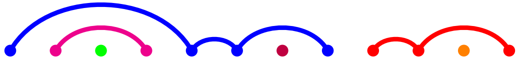


2 6 5 12 9 10 7 11 3 4 1 8

in terms of parking function:

- - - - - - - - - - - -

Posets of noncrossing 2-partitions in terms of parking functions



1 2 3 2 1 1 7 1 9 9 11 9

2 6 5 12 9 10 7 11 3 4 1 8

in terms of parking function:

- 1 - - - - - - 1 1 1 -

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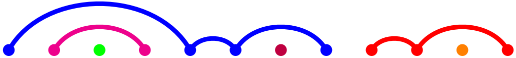


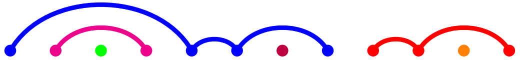
1 2 3 2 1 1 7 1 9 9 11 9

2 6 5 12 9 10 7 11 3 4 1 8

in terms of parking function:

- 1 - - - 2 - - 1 1 1 2

Posets of noncrossing 2-partitions in terms of parking functions



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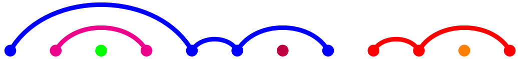

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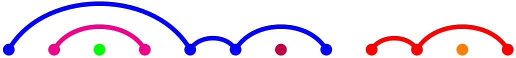
- 1 - - 3 2 7 - 1 1 1 2

Posets of noncrossing 2-partitions in terms of parking functions



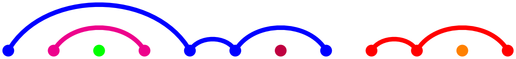
1 2 3 2 1 1 7 1 9 9 11 9

2 6 5 12 9 10 7 11 3 4 1 8

in terms of parking function:

- 1 9 9 3 2 7 9 1 1 1 2

Posets of noncrossing 2-partitions in terms of parking functions



1 2 3 2 1 1 7 1 9 9 11 9

2 6 5 12 9 10 7 11 3 4 1 8

in terms of parking function:

11 1 9 9 3 2 7 9 1 1 1 2

Parking posets (with M. Josuat-Vergès and L. Randazzo)

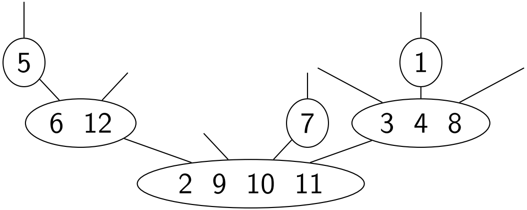
Outline

Parking trees

Definition

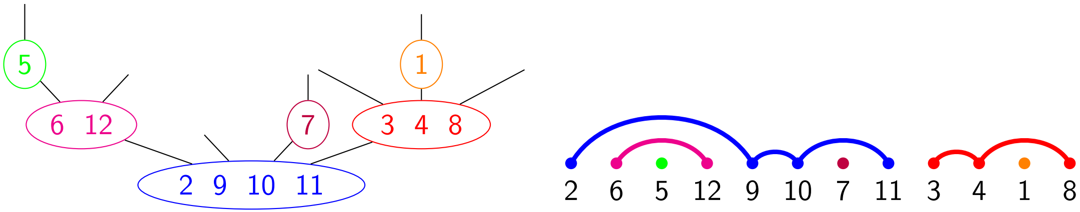
A parking tree on a set L is a rooted plane tree $T = (V, E, r)$ such that:

- $V \in \Pi_L$,
- $v \in V$ has $|v|$ children.



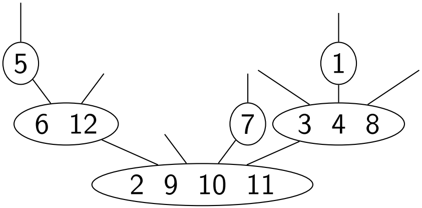
Why parking ?

Bijection between noncrossing 2-partition and parking trees



Coincides with noncrossing parking spaces of Armstrong–Reiner–Rhoades

Functional equation for parking trees



Proposition (DO, Josuat-Vergès, Randazzo, 22)

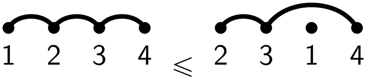
$$\mathcal{P}_f = \sum_{p \geqslant 1} \mathbb{E}_p \times (1 + \mathcal{P}_f)^p$$

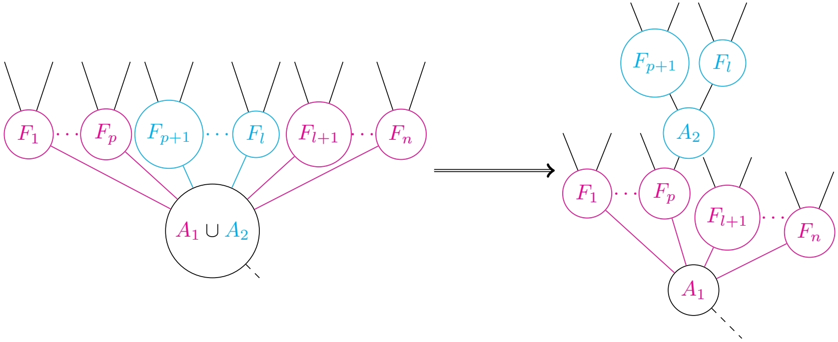
$$\mathcal{P}_f = \sum_{p \geqslant 1} \frac{x^p (1 + \mathcal{P}_f)^p}{p!} = \exp\left(x(1 + \mathcal{P}_f)\right)$$

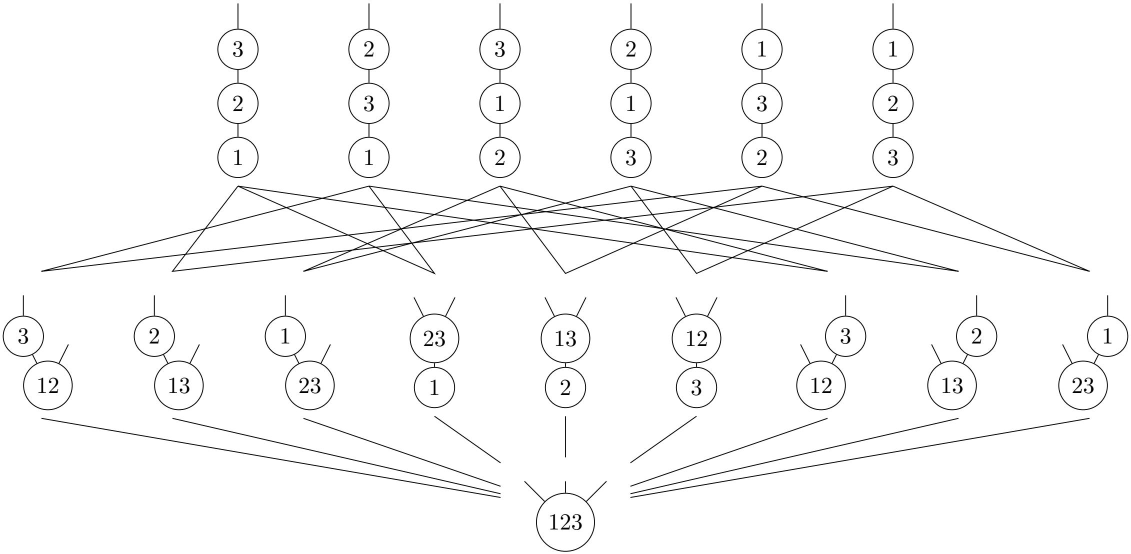
Noncrossing 2-partitions poset

Covering relation in Π^2 : merge parts and rearrange labels to respect the increasing condition

Example :





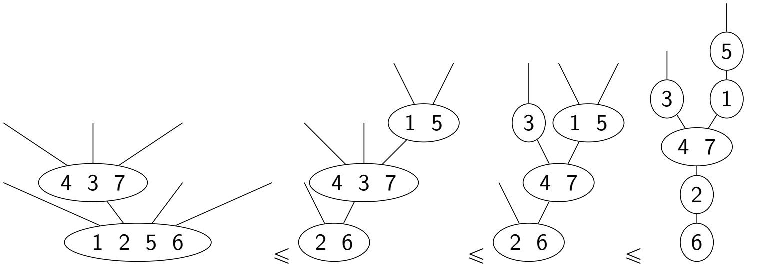

3 2 3 2 1 1
2 3 1 1 3 2
1 1 2 3 2 3
3 2 1 23 13 12 3 2 1
12 13 23 1 2 3 12 13 23
123

Results

Previously known results

Edelman computed the zeta polynomial and Whitney number of the second kind (number of elements by rank) in 1980 and Rhoades computed the zeta character in 2014 (section 8).

- This poset is a lattice.
- When restricting to right combs, get the face poset of the permutohedron.
- New criterion to prove shellability !
- Enumeration of (weak) k-chains, hence computation of Euler characteristic



Proposition (DO, Josuat-Vergès, Randazzo, 22)

$$\mathcal{C}^{l}_{k,t} = \sum_{p \geqslant 1} \mathcal{C}^{l,p}_{k-1,t} \times \left(t\mathcal{C}^{l}_{k,t} + 1\right)^{p}$$

- *Chains* $\phi_1 \leqslant \cdots \leqslant \phi_k$ *in* ${\Pi^2}_n$ *are in bijection with k-parking trees.*
- *The number of chains* $\phi_1 \leqslant \cdots \leqslant \phi_k$ *in* ${\Pi^2}_n$ *where* $\mathrm{rk}(\phi_k) = \ell$ *is:*

$$\ell! \binom{kn}{\ell} S_2(n, \ell + 1).$$

- *Hence, the Whitney number of the first kind are given by:*

$$w_\ell(2NCP_n) = (-1)^{\ell} \ell! \binom{n + \ell - 1}{n} S(n, \ell + 1)$$

More on characters

Proposition (DO, Josuat-Vergès, Randazzo, 22)

The character on the unique non trivial homology group of the augmented poset is:

$$\sigma \mapsto (-1)^{n-z(\sigma)}(n-1)^{z(\sigma)-1}$$

(up to a sign, prime parking functions !)

Proposition (DO, Dupont, 25+)

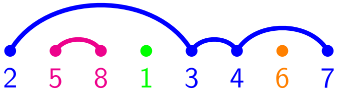
- *The species of noncrossing 2-partitions is an operadic poset species: there is an operadic structure on* $\bigoplus_{m\in\max(2NCP(n))} \tilde{H}^{n-3}([\hat{0}, m])$, *with a preLie operations.*
- *Moreover the underlying species is isomorphic to the direct sum of* C_n *copies of the regular representation.*
- *The vector space spanned by prime parking functions is a left operadic module for this operad.*

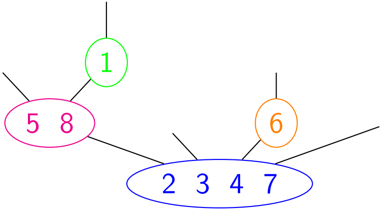
Tamari-parking posets (with M. Josuat-Vergès and H. Han)

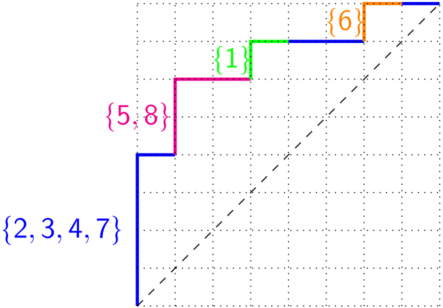
Outline

1. From partition posets to parking posets
2. Parking posets (with M. Josuat-Vergès and L. Randazzo)
3. Tamari-parking posets (with M. Josuat-Vergès and H. Han)
    - Tamari lattices
    - Tamari-parking posets

Noncrossing 2-partitions, labelled Dyck path and parking functions







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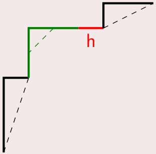
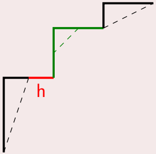
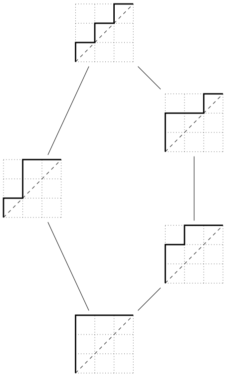
Tamari lattices [Tamari, 1951] on parking trees

Definition

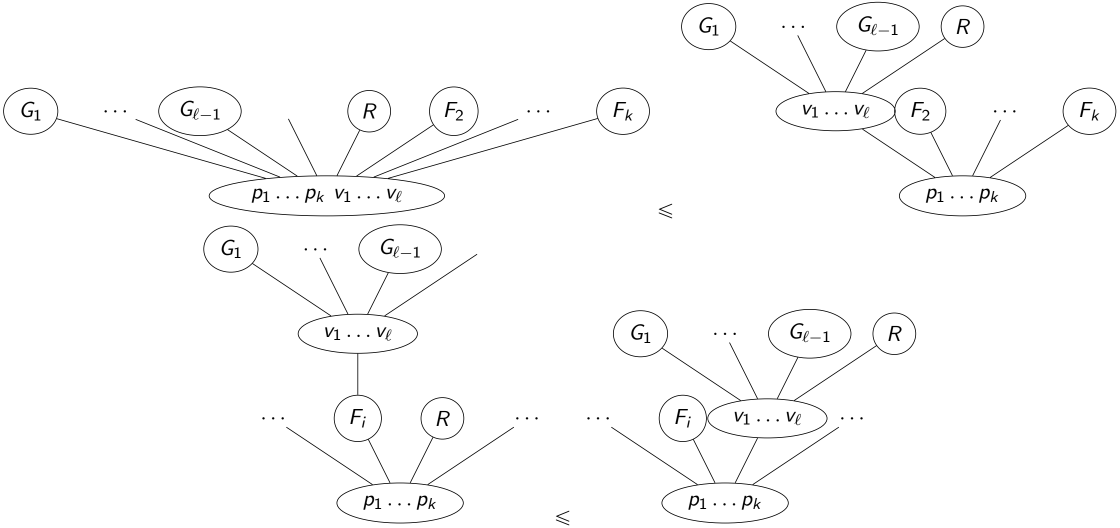
$C =$ 

$\lessdot$ 

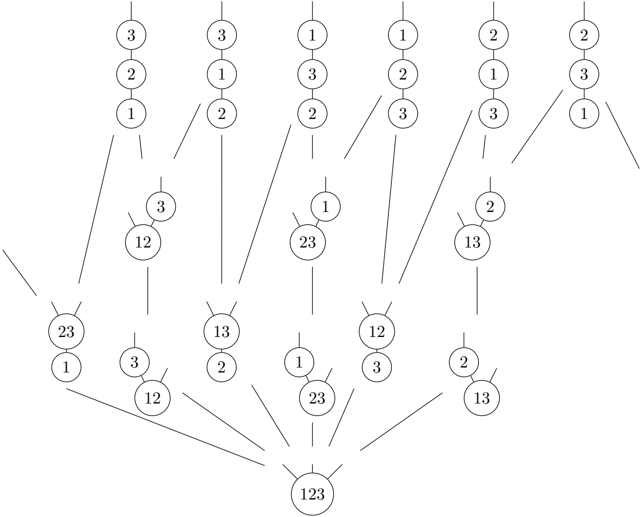
$= D$



Covering relations on Tamari-Parking posets



Tamari-parking poset



Known results and open questions

Proposition (Chapuy–Bousquet-Mélou–Préville-Ratelle, 12)

Say that two intervals in Tamari-parking posets are isomorphic if they have the same minimum element and maximal elements of the same shape.
The number of class of isomorphisms of intervals is given by:

$$2^n(n+1)^{n-2}.$$

The action of the symmetric group on these isomorphisms class of intervals are likely to be, up to a sign, the same as the one on the space of diagonal coinvariants in three sets of n variables.

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The number of intervals in the Tamari-Parking posets and augmented Tamari-Parking posets are:

n	1	2	3	4	5	6
TP_n	1	5	52	855	19 521	574 498
$\hat{TP}_n$	1	9	69	981	20 818	591 306

More on the topology of the Tamari-Parking poset

Conjecture (DO)

Augmented Tamari-parking posets are homotopic to a sphere.

Proposition (H. Han, 24)

Tamari-parking posets are lattices. They are neither EL-shellable nor CL-shellable.

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What to bring home

- Poset cohomologies are fascinating.
- They unveil new operadic structures on nice representations.
- Don't hesitate to bring me your own favourite poset !

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Thank you for your attention !